

# Appendix A: The Standard Hourly Auction Problem

## A1 Definitions

### *Indices:*

$z$ : index denoting aggregate busses (geographic and foreign country zones).  $z = 1, 2, \dots, N$  where  $N$  is defined below.

$i, j$ : indices used to denote aggregate busses between which there exists a power exchange interface.  $i, j \in \{1, 2, \dots, N\}$ .

$\alpha$ : index used to denote some (real, aggregate, or virtual) intra-zone transmission line over which real power flow is limited for various reasons, such as a thermal real power transmission limitation or a stability induced limitation.  $\alpha = 1, 2, \dots, M$ .

$N$ : The total number of aggregate busses modeled. In the enhanced production grade model, the value of  $N$  is not expected to deviate significantly from 20.

$M$ : The total number of (real, aggregate, or virtual) intra-zone transmission lines monitored against congestion. It is expected that the value of  $M$  will not deviate significantly from 10.

$k_c$ : index denoting a consumption bid,  $k_c = 1, 2, \dots, K_C$ . It is expected that  $K_C$  will be originally small (of the order of  $N$ ) and eventually grow to the order of thousands.

$k_g$ : index denoting a generation offer  $k_g = 1, 2, \dots, K_G$ . It is expected that  $K_G$  will not exceed the order of a few thousands.

### *Input Variables:*

$N, K_C, K_G$ : number of aggregate busses, consumption bids, and generation offers, including must run and bilateral contracts.

$C_{ji} = C_{ij}$ :  $\neq 0$  if aggregate bus  $i$  is connected with aggregate bus  $j$ , 0 otherwise. In the enhanced production grade model interconnection topology, it is expected that there will be no more than  $N \times (N-1)$  non zero  $C_{ij}$  values for  $i \neq j$ , and  $i, j \in \{1, 2, \dots, N\}$ .  $C_{ij}$  values are given inputs.

$PV_{k_g}, QOV_{k_g}$ : price-quantity (loss adjusted) pair associated with generation offer  $k_g$  for all  $k_g = 1, 2, \dots, K_G$ . Multiple offers associated with the same offer price are ranked according to a priority assigned to each offer.

$QVMIN$ : minimum generation quantity accepted if a generation offer is accepted at all.

$PA_{k_c}, QOA_{k_c}$ : price-quantity (loss adjusted) pair associated with consumption bid  $k_c$ , for all  $k_c = 1, 2, \dots, K_C$ . Multiple bids associated with the same bid price are ranked according to a priority assigned to each bid.

$MAXF_{ij}$ : Maximum flow allowed over the interconnection from aggregate bus  $i$  to aggregate bus  $j$ , for all  $i \neq j$ , and  $i, j \in \{1, 2, \dots, N\}$  such that  $C_{ij} \neq 0$ .

$S_{ij}^z$ : Contribution of one MW of net injection into aggregate bus  $z$  to the real power flow over the inter-zone power exchange interface connecting zone  $i$  to zone  $j$ . These are calculated from the  $C_{ij}$  coefficient inputs reflecting the appropriate impedance values when loops are present.

$A_\alpha^z$ : Contribution of one MW of net injection into aggregate bus  $z$  to the real power flow over some (real, aggregate, or virtual) intra-zone transmission line  $\alpha$ . This input variable is expected to be provided for all  $z = 1, 2, \dots, N$  and  $\alpha = 1, 2, \dots, M$ . Since this transmission line is internal to an aggregate bus/zone, it is not explicitly modeled as a power exchange interface. This (real, aggregate, or virtual) intra-zone transmission line is included in the power system model coupling various generators in the same constraint. This constraint is associated with the fact that real power flow over the (real, aggregate, or virtual) transmission line  $\alpha$  is limited for various reasons such as a thermal real power transmission limitation or a stability induced limitation.

$b_\alpha$ : Maximum value of allowable power flow over (real, aggregate, or virtual) transmission line  $\alpha$  for all  $\alpha = 1, 2, \dots, M$ .

### **Output Variables:**

$QV_{k_g}, QA_{k_c}$  loss adjusted accepted generation offer and consumption bid quantities for all  $k_g, k_c$ .

$\rho_z$ : Day Ahead Market clearing price in aggregate bus  $z$ .

$\lambda$ : Dual variable value (Lagrange multiplier) associated with the energy balance constraint 4.

$\mu_{ij}$  for all for all  $i \neq j$ , and  $i, j \in \{1, 2, \dots, N\}$  such that  $C_{ij} \neq 0$ : Dual variable values (Lagrange multipliers) associated with the inter-zone power exchange interface constraints 5 of section A2. Note that  $\mu_{ij}$  will be always zero when inter-zone power exchange interface from aggregate busses  $i$  to aggregate bus  $j$  is not binding. For a similar reason, at least one of  $\mu_{ij}$  or  $\mu_{ji}$  will be always zero.

$\nu_\alpha$  for all  $\alpha = 1, 2, \dots, M$ : Dual variable values (Lagrange multipliers) associated with the intra-zone transmission constraints 6 of section A2. Note that if the associated constraint 6 is not binding, then  $\nu_\alpha = 0$ .

$\rho_z = \lambda - \sum_{ij \text{ s.t. } C_{ij}=1} \mu_{ij} \cdot S_{ij}^z - \sum_{\alpha=1,2,\dots,M} \nu_\alpha \cdot A_\alpha^z$  for all  $z = 1, 2, \dots, N$ : Market clearing price in

aggregate bus (or zone)  $z$ .

$\rho_{\text{SenzaVincoliScambio}}$ : National Day Ahead Market clearing price in the absence of inter-zone power exchange or intra-zone transmission constraints. This is obtained by solving the mathematical problem in A2 in the absence of congestion constraints.

Slack on inter zone power exchange interfaces for use in the reserve market auction.

## A2 Mathematical Optimization Problem Employed to Determine Offer/Bid Acceptance and Clearing Prices

STANDARD:

Objective Function: Maximize Consumer plus Producer Surplus

$$1) \quad \text{Max}_{QA_{k_c}, QV_{k_g}} \left\{ \sum_{k_c=1}^{K_C} PA_{k_c} \cdot QA_{k_c} - \sum_{k_g=1}^{K_G} PV_{k_g} \cdot QV_{k_g} \right\}$$

Subject to constraints:

$$2) 0 \leq QA_{k_c} \leq QOA_{k_c} \text{ for all } k_c \in \{1, 2, \dots, K_C\}$$

Continuous consumption bid quantity constraint.

$$3) 0 \leq QV_{k_g} \leq QOV_{k_g} \text{ for all } k_g \in \{1, 2, \dots, K_G\}$$

Generation offer capacity constraint without a minimum acceptable quantity.

$$4) \quad \sum_{k_c=1}^{K_C} QA_{k_c} = \sum_{k_g=1}^{K_G} QV_{k_g} \quad \text{Energy balance constraint.}$$

$$5) \sum_{z=1}^N S_{ij}^z \left[ \sum_{k_g \in \text{aggr bus } z} QV_{k_g} - \sum_{k_c \in \text{aggr bus } z} QA_{k_c} \right] \leq MAXF_{i,j} \quad \text{for all } i, j \in \{1, 2, \dots, N\} \text{ s. t. } C_{ij} \neq 0, i \neq j$$

$$6) \sum_{z=1}^N A_{\alpha}^z \left[ \sum_{k_g \in \text{aggr bus } z} QV_{k_g} - \sum_{k_c \in \text{aggr bus } z} QA_{k_c} \right] \leq b_{\alpha} \quad \text{for all } \alpha = 1, 2, \dots, M.$$

Congestion monitoring of all potentially binding (real, aggregate, or virtual) intra-zone transmission constraints.

## Appendix B: The Hourly Auction Problem with Uniform Purchase Price and Zonal Sell Prices

We first present the basic formulation then introduce additional complications due to particular implementation requirements. The hourly auction problem with uniform purchase price  $P^*$  and zonal prices  $P^z$  is given as follows:

UPPO:

$$1) \quad \text{Max}_{QA_{k_c}, QV_{k_g}} \left\{ \sum_{k_c=1}^{K_C} PA_{k_c} \cdot QA_{k_c} - \sum_{k_g=1}^{K_G} PV_{k_g} \cdot QV_{k_g} \right\}$$

Subject to constraints:

2a) for all  $k_c \in \{1, 2, \dots, K_C\}$

$$0 \leq QA_{k_c} \leq QOA_{k_c} \text{ if } PA_{k_c} = P^* \quad (\text{same as constraint 2 in standard problem})$$

$$QA_{k_c} = 0 \text{ if } PA_{k_c} < P^* \quad (\text{bids below uniform price rejected completely})$$

$$QA_{k_c} = QOA_{k_c} \text{ if } PA_{k_c} > P^* \quad (\text{bids above uniform price accepted completely})$$

3a) for all  $k_g \in \{1, 2, \dots, K_G\}$

$$0 \leq QV_{k_g} \leq QOV_{k_g} \text{ if } PV_{k_g} = P^z \quad (\text{same as constraint 3 in standard problem})$$

$$QV_{k_g} = 0 \text{ if } PV_{k_g} > P^z \quad (\text{offers above zonal price rejected completely})$$

$$QV_{k_g} = QOV_{k_g} \text{ if } PV_{k_g} < P^z \quad (\text{offers below zonal price accepted completely})$$

$$4) \quad \sum_{k_c=1}^{K_C} QA_{k_c} = \sum_{k_g=1}^{K_G} QV_{k_g} \quad (\text{same as constraint 4 in standard problem})$$

$$5) \quad \sum_{z=1}^N S_{ij}^z \left[ \sum_{k_g \in \text{aggr bus } z} QV_{k_g} - \sum_{k_c \in \text{aggr bus } z} QA_{k_c} \right] \leq MAXF_{i,j} \quad \text{for all } i, j \in \{1, 2, \dots, N\} \text{ s. t. } C_{ij} \neq 0, i \neq j$$

(same as constraint 5 in standard problem)

$$6) \sum_{z=1}^N A_{\alpha}^z \left[ \sum_{k_g \in \text{aggr bus } z} QV_{k_g} - \sum_{k_c \in \text{aggr bus } z} QA_{k_c} \right] \leq b_{\alpha} \quad \text{for all } \alpha = 1, 2, \dots, M.$$

(same as constraint 6 in standard problem)

$$7a) P^* \sum_{k_c=1}^{K_c} QA_{k_c} - \sum_{z=1}^N P^z * \cdot \left( \sum_{k_g \in \text{aggr bus } z} QV_{k_g} \right) = 0$$

Equality between payments and revenues (cost recovery of marginal generation cost only)

or

$$7b) P^* \sum_{k_c=1}^{K_c} QA_{k_c} - \sum_{z=1}^N P^z * \cdot \left( \sum_{k_c \in \text{aggr bus } z} QA_{k_c} \right) = 0$$

Equality between payments and revenues (cost recovery including hourly transmission rent)

Either 7a or 7b is imposed.

Unfortunately, this new formulation is non linear and hence cannot be solved as a single Linear Program as was the case with the standard formulation given in Appendix A.

### UPPO search procedure

Let  $\{P_b^*\}$  denote the sequence, in ascending order, of bid price breaks of the demand curve. In general  $\{P_b^*\} = \{0, \dots, P_b^{\max} \leq \text{BID-VOLL}\}$  where BID-VOLL denotes the system imposed maximum bid price.

Then for each  $P^*$  in  $\{P_b^*\}$  we can evaluate a trial solution to the UPPO problem given above as follows:

- (1) Write constraint 2a as follows using the current  $P^*$ 
  - a.  $QA_{k_c} = 0$  if  $PA_{k_c} < P^*$
  - b.  $QA_{k_c} = QOA_{k_c}$  if  $PA_{k_c} \geq P^*$
- (2) Use constraint 3 from the STANDARD problem instead of 3a in UPPO
- (3) Drop constraint 7 (cost recovery balance)
- (4) Solve the resulting Linear Program
- (5) Evaluate cost recovery (using either 7a or 7b)

If cost recovery is balanced then this is a feasible candidate solution

Otherwise, we continue to explore at  $P^*$  as follows:

- (1) Replace 1b constraint above with following:
  - a.  $QA_{k_c} = QOA_{k_c}$  if  $PA_{k_c} > P^*$
  - b.  $\sum_{PA_{k_c}=P^*} QA_{k_c} = T \cdot \sum_{PA_{k_c}=P^*} QOA_{k_c}$  where  $T \in [0, 1]$
- (2) Solve the resulting Parametric Linear Program
- (3) Evaluate cost recovery at each break point of T

Any break point solution that balances cost recovery is a candidate feasible solution to UPPO. Additional candidate feasible solutions at  $P^*$  are found as follows:

- (1) A candidate feasible solution can be found at a break point of the Parametric Linear Program by interpolation of the zonal prices  $P^z$  which change discontinuously at these break points.
- (2) A candidate feasible solution can be found between two subsequent break points of the Parametric Linear Program by interpolation of the problem control variables ( $QA_{k_c}$  and  $QV_{k_g}$ ) which change continuously and linearly in this interval.
- (3) A candidate feasible solution can be found at a uniform purchase price between the current  $P^*$  and the next highest  $P^*$  in  $\{P_b^*\}$  by interpolation of  $P^*$  in the cost recovery balance equation (7a or 7b). In this case we are at a vertical jump in the demand curve and the candidate UPPO solution remains constant.

Applying the above search procedure on the sequence  $\{P_b^*\}$  the optimal uniform purchase price is the one associated with the candidate feasible solution with the maximum value (i.e. value of the objective function).

### **Demand rationing due to transmission limitations**

Bid quantities are fixed at their upper limits at various phases of the above search procedure. This can lead to infeasible problem formulations due to transmission constraints. In order to alleviate this situation the following modifications are required:

- (1) The basic problem is modified by introducing a “dummy” positive cost generation offer (PCGen) in each zone valued at VOLL and with “infinite” capacity (the total demand quantity is used). A separate “dummy” zero cost generation offer (ZCGen) at zero price and an infinitesimal capacity ( $\epsilon$ ) is also added to the problem for each zone. These ZCGen generation offers are used to correctly identify zonal prices in all possible situations (see discussion on demand and supply curve crossing conditions and related issues in standard formulation).
- (2) The STANDARD problem with the PCGen and ZCGen generation offers is solved. In the solution to this problem, any zone that has scarcity (unfulfilled demand due to transmission constraints) will have a zonal price equal to VOLL.

- (3) If scarcity is identified at step (2) a new Linear Program that maximizes the value of supported demand subject to transmission constraints is solved in order to determine the appropriate demand curtailment. This means that offers are dropped from the objective function and the PCGen generation offers are removed in order to correctly account for scarcity (otherwise they would supply the excess demand in each scarce zone). Finally, non-regular bids are dropped. The resulting objective function is as follows:

$$\text{Max}_{QA_{k_c}} \sum_{k_c=1}^{K_c} PA_{k_c} \cdot QA_{k_c}$$

Where  $QA_{k_c} = 0$  if  $k_c$  is a non-regular bid.

- (4) The bid quantities for demand bids that are curtailed are now modified prior to the start of the UPPO search procedure. The bid quantity ( $QOA_{k_c}$ ) for the last (at least partially) accepted bid (in bid priority order) is changed to  $QA_{k_c} + N\varepsilon$  where  $QA_{k_c}$  is the accepted quantity for this (marginal curtailed) bid  $k_c$  in the solution of step (3). All other  $QOA_{k_c}$  are set to  $QA_{k_c}$ . This insures that binding transmission constraints and VOLL generators supplying scarce zones remain active at the start of the subsequent UPPO search procedure.
- (5) The regular generation offers and the PCGen generation offers are now reinstated prior to the UPPO search procedure. The price of a PCGen generator in a scarce zone is set at the price of the marginal curtailed bid in that zone. The capacities of all PCGen generation offers are set to  $(N + 1)\varepsilon$ . Finally, non-regular bids are reinstated.

The regular UPPO search procedure can now be applied with the PCGen generators in scarce zones competing with regular generation offers.

### **Non-regular bids**

Demand bids from external zones and domestic bids from units of type “BOTH” are subject to zonal prices instead of the uniform purchase price. Such bids are called non-regular in the above discussion. This requirement is taken into account by using constraint 2 from the STANDARD formulation for all non-regular bids at all stages of the problem solution procedure including initial scarcity detection and the UPPO search procedure. In addition, demand curtailment after scarcity detection is not applied to non-regular bids. Moreover, during the UPPO search procedure, all cost recovery calculations use the zonal price  $P^z$  \* instead of  $P^*$  when accounting for payments from accepted quantities of non-regular bids.

### **Stopping rule**

The UPPO search procedure can be very time consuming when the problem size is large. Consequently, the following stopping rule is used in order to end the search procedure once at least one feasible candidate solution has been found:

At a candidate feasible solution, if for each zone  $z$  the following holds then we declare the current best solution as the optimal UPPO solution:

The price of the lowest accepted bid in zone  $z$  is not lower than  $P^z *$



# **Appendix C: UPPO Algorithm Complications Arising When Transmission Constraints Do not Allow All Bids to be Served Under an Appropriately Low Uniform Price: Problem Description and Proposed Solution**

## **C1 Outline**

We describe here the problem that made desirable a modification of the Demand Side Uniform Price (UPPO) Energy Market Auction implementation Algorithm. To deal with this problem, it is necessary, albeit only under rare occasions, to violate the “No Surprise” Assumption – i.e. the assumption that no bids that come in at a price bigger than the Uniform Demand Price,  $P^*$ , are rejected. In addition to the problem description, we present the costs and the undesirable market behavior that would result if we insisted in meeting the “No Surprise” Assumption under absolutely all circumstances. Finally, we present the solution adopted, describe the avoided costs, but also its drawbacks which we also attempt to evaluate.

## **C2 Problem Description**

UPPO specifications require that all national zone bids (i.e., excepting bids in foreign zones) are charged a uniform price, while generation offers are paid according to geographically differentiated zonal prices that reflect the marginal cost of meeting an incremental MWh of load at each zone. In addition, the UPPO specifications require that the “No Surprise” Assumption is observed, namely that

(i) no bids are rejected if the associated bid price exceeds the Uniform Demand Side Price,  $P^*$ , and moreover

(ii) no bids are accepted if the associated bid price is smaller than  $P^*$ . This assumption is desirable from a social welfare and fairness point of view.

A practical – i.e., computationally tractable given time constraints – algorithm was designed as follows:

- Start with a tentative  $P^*$  value and determine those bids and offers that must be accepted for each  $P^*$  value in order to maximize social welfare, i.e. maximize the sum of all consumer and producer surpluses while observing transmission constraints.
- Search over all possible  $P^*$  values starting from a low value – practically zero -- and increasing it monotonically until gross revenue from bids equals total payments to generators, or, if transmission rent recovery is desired, until gross revenues cover both generator payments plus transmission rent.
- Continue until all such zero net revenue uniform prices are identified – if more than one exists – and select the one that is associated with the highest social welfare value.

The algorithm described above works well and can be even speeded up to avoid searching for additional zero net revenue Uniform Prices that can be predicted to yield lower social welfare than those already identified<sup>3</sup>. However, it may yield solutions that are clearly undesirable when a certain type of transmission constraints is active for low tentative  $P^*$  values. In fact, there are two types of transmission constraints that are relevant to this discussion:

1. Transmission constraints that may be met by out of merit dispatch of generation offers, i.e., by accepting offers in some zones that with higher offer price than offers in other zones, and
2. Transmission constraints that can be met only with curtailing or rejecting bids in certain zones.

In the rare occasion that transmission constraints of type (2) are active, the UPPO algorithm described above is difficult to implement. Although the implementation of the original UPPO algorithm is indeed mathematically and computationally possible, refusal to relax part (i) of the “No surprise Assumption”, results in final  $P^*$  values that are undesirable in ways described below.

### **C3 Cost of Refusing to Relax the No Surprise Assumption**

The major cost of adhering to the original UPPO specifications with no exception is described next.

Note that satisfaction of part (i) of the “No Surprise” Assumption, when type (2) transmission constraints are present, requires that the acceptable uniform price,  $P^*$ , must equal or exceed the bid price of those bids that are infeasible under the active transmission constraints.

If these bid prices are high, increasing  $P^*$  to that high level will ration – i.e., reject – all other bids with lower bid prices, in whichever zone they happen to be. In fact it is possible, that a VOLL price bid, i.e., a bid that is a must meet bid, can not be possibly met due to a transmission constraint. Such a situation is not common, but it may arise, in fact it does arise whenever regional blackouts are made necessary and are resorted to. Honoring part (i) of the “No Surprise” assumption in this case would mean that the Uniform price  $P^*$  would be set equal to VOLL and as such generalize a regional blackout to a national blackout by postulating that all bids ought to be rejected.

Less severe, though still undesirable, nation wide bid curtailment and welfare loss may take place if a high value of  $P^*$  -- not VOLL, but high enough – were to be necessitated by the conjunction of type (2) transmission constraints and the desire to maintain part (i) of the “No Surprise” assumption in some zone.

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<sup>3</sup> See item 2 in October 2, 2002 Discussion in section A6

To avoid the welfare loss scenarios summarized above and described further with concrete examples in the October 30-31, 2002 discussion presented in Section C6, the following modification of the UPPO algorithm was agreed upon and implemented.

#### **C4 UPPO Modifications Adopted**

1. Whenever transmission constraints of type (2) are present that render it physically impossible to meet all of the bids submitted, then:

1.1 A list of bids that are candidates for quantity rationing under the uniform price rule are identified by solving an Optimization Problem called FEASIBILITYOPTIMIZATION which maximizes the value of submitted bids subject to transmission and generation offer capacity constraints. Bids not selected in the solution of the above FEASIBILITYOPTIMIZATION are tentatively truncated and form the list of quantity rationing candidates mentioned above. Bids in this list, may be rejected in violation of part (i) of the "No Surprise" Assumption. However, they are only candidates for this violation. It is possible that the final solution may not violate the "No Surprise" Assumption for all or for some of the bids in the candidate list.

1.2 The regular UPPO algorithm is then applied to determine the best (i.e. highest social utility) uniform price,  $P^*$ , that meets the zero net revenue requirement.

1.3 Some (possibly all) of the bids rationed in step 1.1 may end up being "price rationed" if the final uniform price  $P^* \geq$  the associated bid prices. Any remaining truncated bids are considered the result of quantity rationing and present violations of part (i) the "no surprise condition". The justification of such violation is (a) the physical impossibility of meeting these bids, and (b) the reasonable desire/decision to limit the impact of binding transmission constraints to the affected zones.

2. In the event that demand is truncated in step 1 due to transmission constraints, the zonal price in each zone where demand was truncated reflects the price of the marginal bid that was truncated (NOT VOLL). This is enforced by assigning the marginal truncated bid price as the generation cost to a fictitious-infinitesimally-small-capacity-generator in that zone. Fictitious-infinitesimally-small-capacity-generators compete with the cost of generation offers in that zone. If the highest cost generator in that zone has a price higher than the one assigned to the fictitious-infinitesimally-small-capacity-generator of that zone, that generator's cost and not the fictitious generator's cost determines the zonal price when the regular UPPO algorithm execution starts.

3. If transmission constraints indicate that there must be truncation, the FEASIBILITYOPTIMIZATION is solved so as to maximize the value of bids WITHIN Italy. Bids in external zones -- if any -- are treated as responding to zonal prices and are hence subject to price rationing. As a result, no quantity rationing is needed in foreign zone bids. In general bids as well as offers in external zones are treated as decision variables in the UPPO algorithm and are subjected to zonal prices as far as the zero net revenue requirement is concerned. Only domestic bids are charged and price rationed according to the UPPO Uniform Price,  $P^*$ .

## **C5 Description and Evaluation of Rare Occasions of Welfare Optimality Loss, and Drawbacks of Attempting to Guarantee their Absence**

When type (2) transmission constraints are present, although the formation of the list of candidate bids for price rationing through the solution of the FEASIBILITYOPTIMIZATION described above is reasonable, it is possible in the presence of loop forming transmission constraints that a different selection of quantity rationed bids may yield a higher social welfare market clearing. An example demonstrating the existence of such a scenario was in fact constructed and is reported in the discussion dated November 18, 2002 included in section C6.

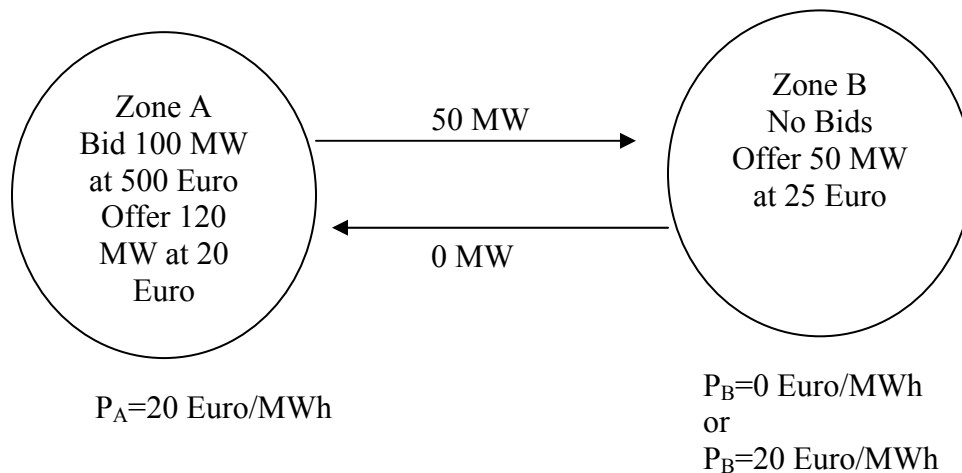
Careful consideration of this scenario, however, resulted in a decision to avoid searching systematically for its detection and the identification of the quantity bid rationing that guarantees social welfare maximization. The rationale of this decision includes:

- The extremely rare and possibly never to be encountered occurrence of the constructed example in a real situation.
- The complexity of the algorithm that would guarantee optimality. In fact, the requisite algorithm would be a combinatorial search algorithm that is likely to be too slow for the relevant time performance constraints.
- The possible loss of welfare is minuscule and certainly smaller than the welfare loss introduced by other fundamental assumptions and model simplifications such as the decision to represent consecutive hourly energy markets as independent when unit commitment considerations indicate a significant inter-temporal coupling.

Section C6 that follows, includes the detailed discussions between GME and TCA that resulted in the UPPO algorithm modifications that allow infrequent but required part (i) “No Surprise” Assumption exceptions summarized above.

## Appendix D: Ambiguities in the Determination of Marginal Cost Prices in Zones with No Bids and Zero Export Interconnection Capacity

In early testing, TCA observed that when export interconnection capacity was set to zero in a zone with no bids, the standard market clearing algorithm returned a zonal price equal to 0. This price is indeed the correct opportunity cost of incremental generation in that zone. However, the marginal cost of incremental demand might be larger than 0. In the example below, due to the zero export constraint, the incremental opportunity cost of generation in zone B is 0 Euro/MWh while the cost of incremental demand is 20 Euro/MWh, the zonal price in Zone A which would satisfy an incremental bid in zone B.



The ambiguity of the marginal cost price in Zone B is caused by the discontinuity of the derivative of cost with respect to net injection in zone B, otherwise known as discrepancy between left and right derivatives. When a phenomenon of this sort occurs, we observe that the cost derivative does not exist. Instead, what exists is a sub-gradient bounded by the left and right derivative. The existence of a sub-gradient in the example above prescribes that  $0 \leq P_B \leq 20$  Euro/MWh, namely that the acceptable price lies inside a range of acceptable prices. In all cases, except for situations with zones featuring zero bids and no allowed exports, the left and right derivatives coincide and the range of prices observed in the above example collapses to a single, unique point.

The TCA algorithm estimates zonal prices as the marginal opportunity cost of incremental generation, namely it uses the derivative of costs with respect to positive net injections (the derivative from the right). In an early version of the market-clearing algorithm, a heuristic rule was implemented that set the zonal price equal to the incremental cost of demand. It was thought at the time that this was preferable from an “aesthetic” point of view. Since zonal prices are irrelevant in the absence of bids and

allowable exports, this adjustment indeed had an aesthetic rather than a practical motivation. With the extended use of the standard algorithm in the Adjustment markets, however, it was realized that situations where one or more zones featured zero bids and no exports were allowed, were quite common. In some of these cases, the heuristic rule gave occasionally incorrect results by erroneously estimating the left derivative as the cost of incremental imports without accounting for zonal offer incremental costs. To avoid what would have been a time consuming algorithmic implementation that would have been required to always determine accurately left and right derivatives, we opted to remove the heuristic rule altogether, and report always the correct right derivative. We therefore report the marginal opportunity cost of incremental generation (i.e. positive injection) as opposed to the sometimes different incremental marginal cost of demand. In conclusion, whenever a range of zonal prices is applicable, the algorithm reports the lower bound of that range.